

The Twenty-One Relations

The φ -Solid's Lattice — Co-Emergent Ternary Algebra

The interactive φ -solid places the twelve vantages of the corpus on the vertices of an **icosahedron**, whose coordinates are $(0, \pm 1, \pm \varphi)$. Each relation below is a mathematical identity **shared by two vantages**, drawn from the corpus in 3.zip. The vertex assignment was chosen by exhaustive search over all 46,080 axis-permutations and orientations to maximize how many relations land on **true icosahedron edges**: **18 of 21** do. The remaining **3** are genuine chords — they connect non-adjacent vertices and render as curved arcs in the interactive.

$\alpha \equiv \Omega \equiv X \equiv \varphi \equiv e^{\pi} \equiv 1/\varphi - \varphi$ every relation is a face of this one identity, shared between two vantages · edge length on the unit solid = 1.0515

ON TRUE ICOSAHEDRON EDGES — 18 of 21

VANTAGE PAIR	SHARED IDENTITY	TYPE	VERTEX DIST.	
1 The Ternary Seed — 5 The Machine $\mathbb{M}_0$	$\mathcal{T}$ is the alphabet of $\mathbb{M}_0 = (\mathcal{T}, \mathcal{J}, \text{Fix})$	• machine / spec	1.0515	EDGE
1 The Ternary Seed — 7 The Prism of Seven	$3 + 3 + 1 = 7$ — the prism built from $\mathcal{T}$	• zero-relation / X	1.0515	EDGE
1 The Ternary Seed — 9 The $D_n(r)$ Lattice	$\varphi^n = \text{Fib}_n \varphi + \text{Fib}_{n-1}$ feeds the lattice magnitude	• golden / φ -ladder	1.0515	EDGE
2 Prismatic Inversion — 5 The Machine $\mathbb{M}_0$	$\mathcal{J}(x) = -x$ is the transition function of $\mathbb{M}_0$	• inversion / axiom	1.0515	EDGE
2 Prismatic Inversion — 3 The Zero Relation	stage one of the pipeline: seed $\rightarrow$ identities	• inversion / axiom	1.0515	EDGE
2 Prismatic Inversion — 7 The Prism of Seven	the inversion stage is the pairing $\mathcal{T} \circ (-\mathcal{T})$	• inversion / axiom	1.0515	EDGE
3 The Zero Relation — 4 Coupling as Origin	$X + 1 = 0$ as the shadow of $\Omega C^2 \equiv e^{\wedge}\{i\pi\}$	• coupling / Ω -phase	1.0515	EDGE
3 The Zero Relation — 12 Traditional Algebra	$x^2 - x - 1 = 0$, solved by the quadratic formula	• zero-relation / X	1.0515	EDGE
4 Coupling as Origin — 12 Traditional Algebra	$e^{\wedge}\{i\pi\} = -1$, recovered by Taylor partial sums	• coupling / Ω -phase	1.0515	EDGE
4 Coupling as Origin — 6 The Co-Emergent Axiom	$\alpha \equiv \Omega$ is the coherence primitive of $\mathfrak{A}$	• inversion / axiom	1.0515	EDGE
6 The Co-Emergent Axiom — 11 The Listing	the listing enumerates $\mathfrak{A}$ line by line	• inversion / axiom	1.0515	EDGE
7 The Prism of Seven — 8 The Octave Return	$7 \rightarrow 8$: completed closure returns at the next scale	• golden / φ -ladder	1.0515	EDGE
8 The Octave Return — 9 The $D_n(r)$ Lattice	the φ -power ladder drives $n = \varphi \cdot \text{Fib}_n \cdot 2^n \cdot \text{Prime}_n$	• golden / φ -ladder	1.0515	EDGE
8 The Octave Return — 12	$\varphi^2 = \varphi + 1$ in ordinary algebra	• golden / φ -ladder	1.0515	EDGE

VANTAGE PAIR	SHARED IDENTITY	TYPE	VERTEX DIST.	
Traditional Algebra				
9 The $D_n(r)$ Lattice — 10 The Machine Spec	$D_n(r)$ emitted as the lattice block of the spec	• machine / spec	1.0515	EDGE
10 The Machine Spec — 5 The Machine $\mathfrak{M}_0$	$\mathfrak{M}_0$ formalized as the executable machine spec	• machine / spec	1.0515	EDGE
10 The Machine Spec — 11 The Listing	the spec compresses to the numbered listing	• machine / spec	1.0515	EDGE
11 The Listing — 12 Traditional Algebra	listing identities checked in float64 arithmetic	• machine / spec	1.0515	EDGE

These 18 relations connect **adjacent vertices** — endpoints one icosahedron edge apart (distance 1.0515 on the unit solid). In the interactive they are straight, colored edges lying exactly on the wireframe: the relation graph very nearly *is* the edge skeleton of the φ -solid.

CHORDS — NOT ON EDGES — 3 of 21

VANTAGE PAIR	SHARED IDENTITY	TYPE	VERTEX DIST.	
1 The Trinary Seed — 3 The Zero Relation	$-1 + 1 = 0$ becomes the closure $X + 1 = 0$	• zero-relation / X	1.7013	CHORD
2 Prismatic Inversion — 6 The Co-Emergent Axiom	the pipeline enumerates the projections of $\mathfrak{M}$	• inversion / axiom	1.7013	CHORD
6 The Co-Emergent Axiom — 8 The Octave Return	$\mathcal{C}(n+1) = \mathcal{C}(\mathcal{C}n)$: the axiom under recursion	• inversion / axiom	1.7013	CHORD

No assignment can place all 21 on edges — the icosahedron has 30 edges but a fixed adjacency structure, and these 3 pairs are related while sitting at **non-adjacent vertices**. Rather than distort the geometry, the interactive renders them honestly as curved arcs bowing outward and labels them “(chord).” Their being chords is informative: 2—6 (pipeline $\leftrightarrow$ axiom) and 6—8 (axiom under recursion) reach across the solid because the Co-Emergent Axiom is the “vantage about vantages,” structurally central rather than locally adjacent; 1—3 (trinity $\rightarrow$ zero-relation) spans the two competing accounts of the seed.

THE SIX ANTIPODAL AXES — $i + (13 - i) = 13$

ANTIPODAL PAIR	CLOSURE	COORDINATES (THROUGH 1 EFF AT ORIGIN)
1 The Trinary Seed — 12 Traditional Algebra	$1 + 12 = 13$	$(+0.000, -0.526, -0.851) \leftrightarrow (+0.000, +0.526, +0.851)$
2 Prismatic Inversion — 11 The Listing	$2 + 11 = 13$	$(+0.000, +0.526, -0.851) \leftrightarrow (+0.000, -0.526, +0.851)$
3 The Zero Relation — 10 The Machine Spec	$3 + 10 = 13$	$(+0.526, +0.851, +0.000) \leftrightarrow (-0.526, -0.851, +0.000)$
4 Coupling as Origin — 9 The $D_n(r)$ Lattice	$4 + 9 = 13$	$(-0.526, +0.851, +0.000) \leftrightarrow (+0.526, -0.851, +0.000)$
5 The Machine $\mathfrak{M}_0$ — 8 The Octave Return	$5 + 8 = 13$	$(-0.851, +0.000, -0.526) \leftrightarrow (+0.851, +0.000, +0.526)$
6 The Co-Emergent Axiom — 7 The Prism of Seven	$6 + 7 = 13$	$(-0.851, +0.000, +0.526) \leftrightarrow (+0.851, +0.000, -0.526)$

Independent of the 21 relations, the numbering places each vantage **diametrically opposite** its partner: vantage i and vantage $13 - i$ sit at antipodal vertices, so the line between them passes through **1_eff** at the center. This is what the interactive's $\mathcal{I}$ operation makes physical — the inversion sends every disc through the origin onto its antipode, and $\mathcal{I} \circ \mathcal{I}$ returns home.

Generated from the same relation table and icosahedron coordinates that drive coemergent_phisolid.html. Source corpus: 3.zip, vantages 1–11 plus the traditional-algebra vantage (12). Distances are on the unit-radius solid.